

Esercizio

Calcolare la soluzione ottima del seguente problema $1 / r_j / \sum h_j$

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

Scelta del rilassamento

Osservazione: ponendo il job 5 in ultima posizione e sommando la massima release date, si ottiene $T_5 = 16 + 4 - 12 = 8$. Quindi $8U_5 \geq T_5$.

Inoltre, essendo $L_j \leq T_j$ il problema

$$1/r_j / \sum L_j$$

è un rilassamento del problema dato. Quindi, $1/r_j, prmp / \sum L_j$ è un rilassamento del problema dato.

Essendo $L_j = C_j - d_j$, il rilassamento è equivalente a

$$1/r_j, prmp / \sum C_j$$

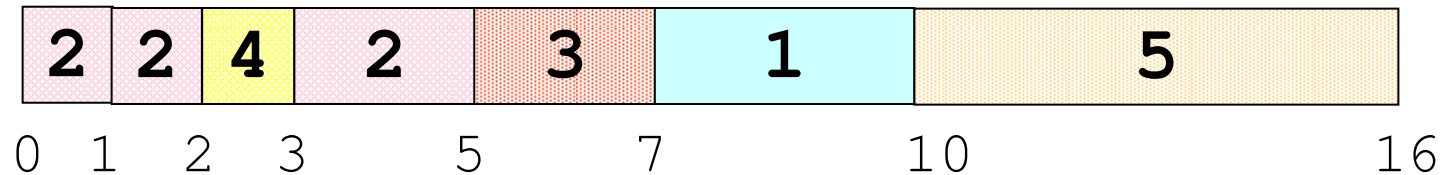
polinomialmente risolubile con la regola **SRPT**

Nodo radice

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

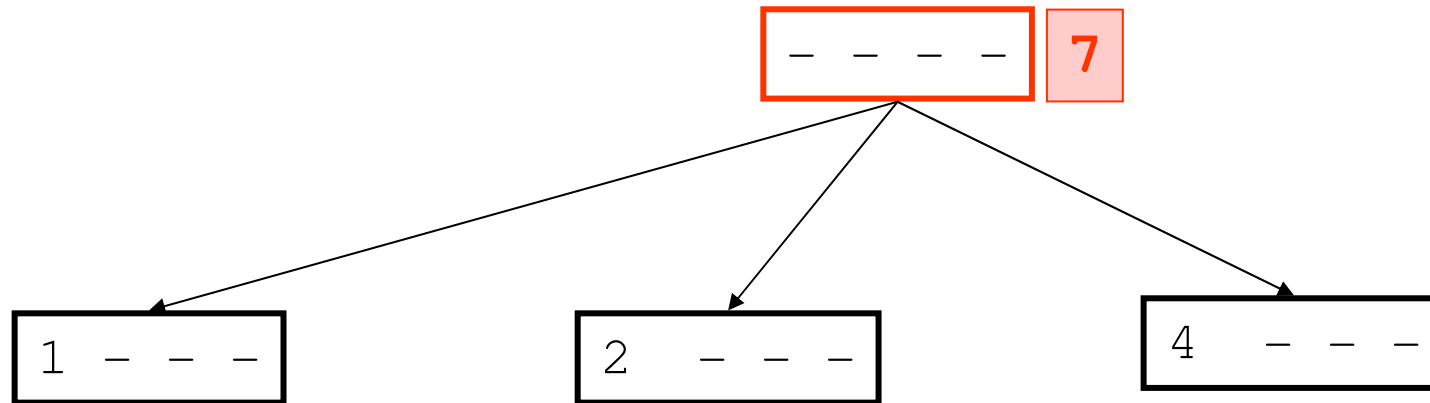
$$\Sigma d_j = 34$$

$$\Sigma L_j = 41 - 34 = 7$$



SRPT costruisce una soluzione non ammissibile di valore 7

Branching



i sottoproblemi $\{3 - - -\}$ e $\{5 - - -\}$ sono eliminati per dominanza.

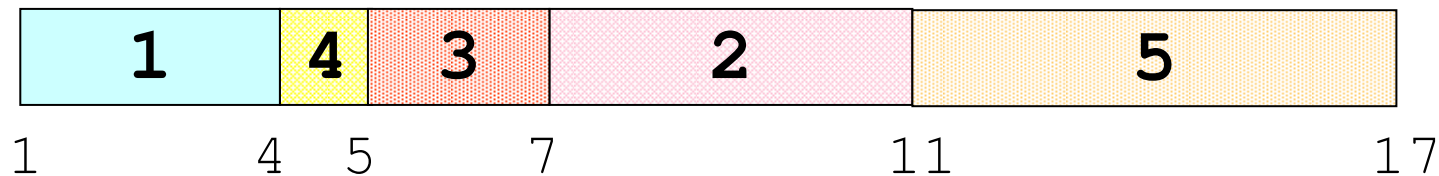
Nodo {1 - - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 44 - 34 = 10$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 5 + 1 + 0 + 8 = 13$$

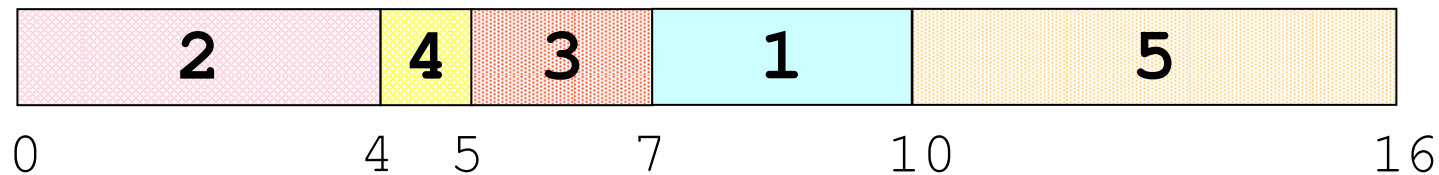
aggiorna ottimo corrente

Nodo { 2 - - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 42 - 34 = 8$$



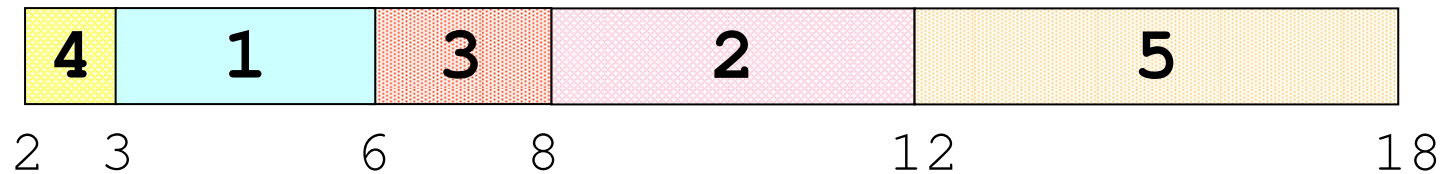
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 1 + 0 + 8 = 14$$

Nodo { 4 - - - }

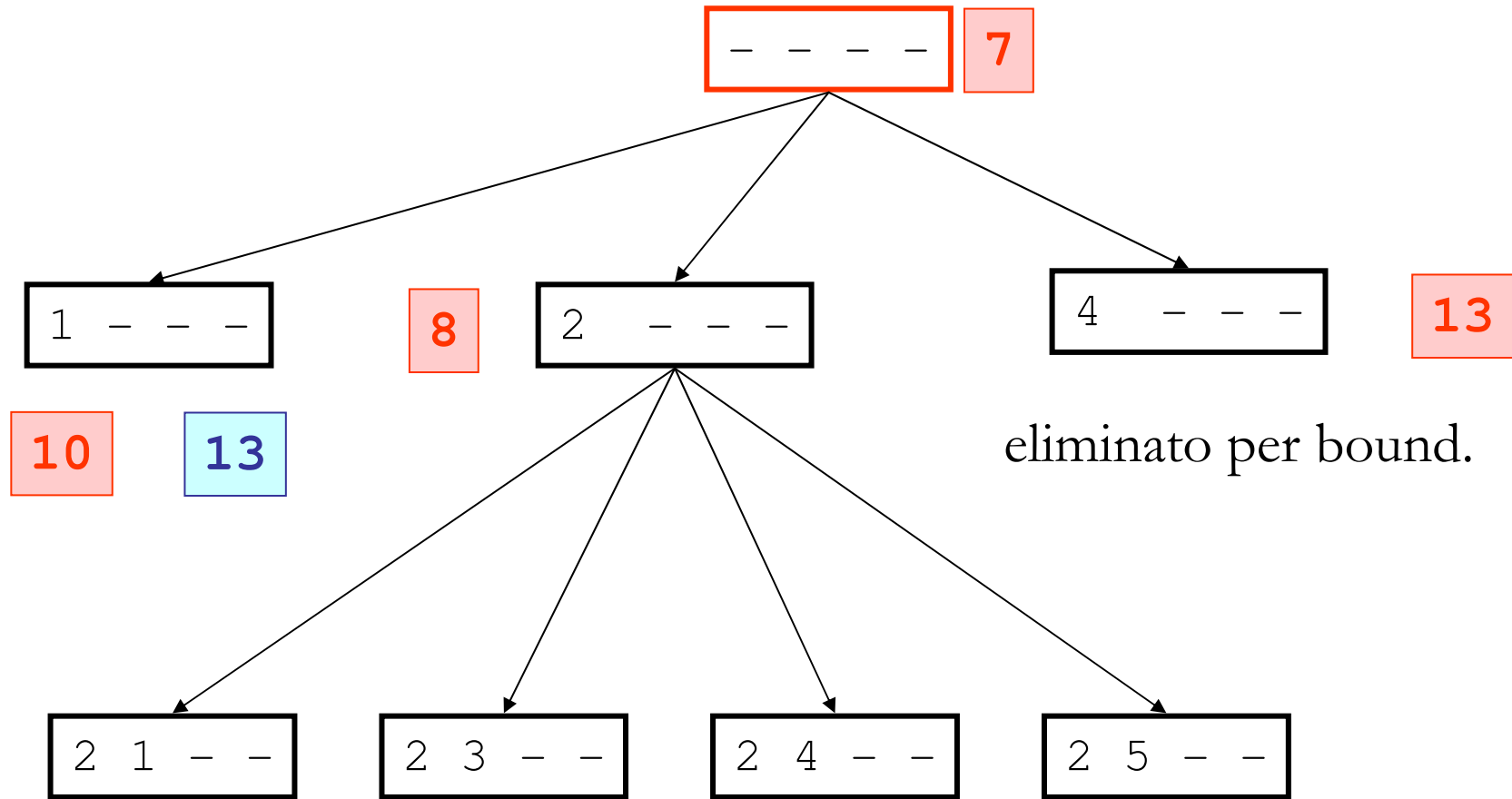
job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 47 - 34 = 13$$



Branching



Visita “bound più promettente”

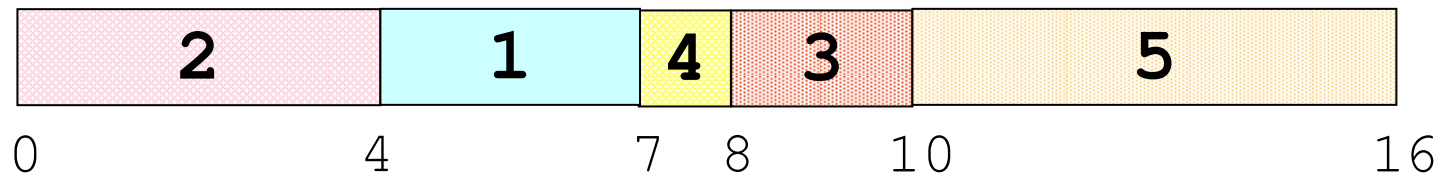
Nodo {2 1 - - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 45 - 34 = 11$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 4 + 3 + 8 = 17$$

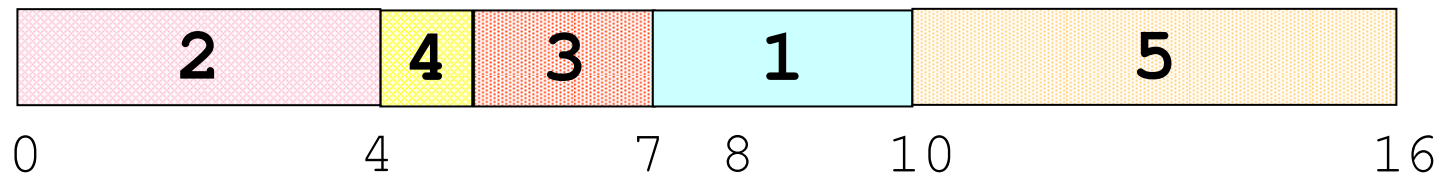
Nodo { 2 4 - - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 42 - 34 = 8$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 1 + 0 + 8 = 14$$

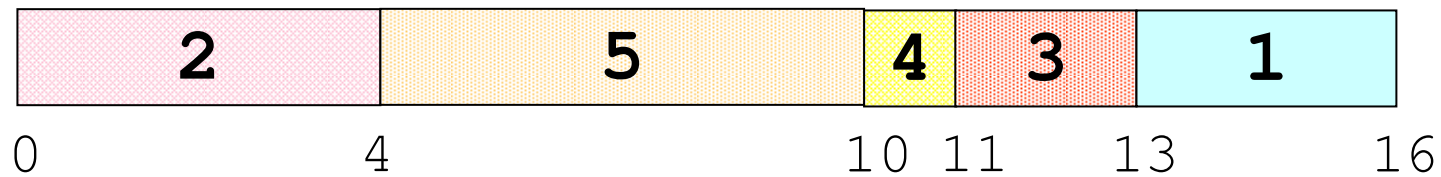
Nodo { 2 5 - - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 54 - 34 = 20$$



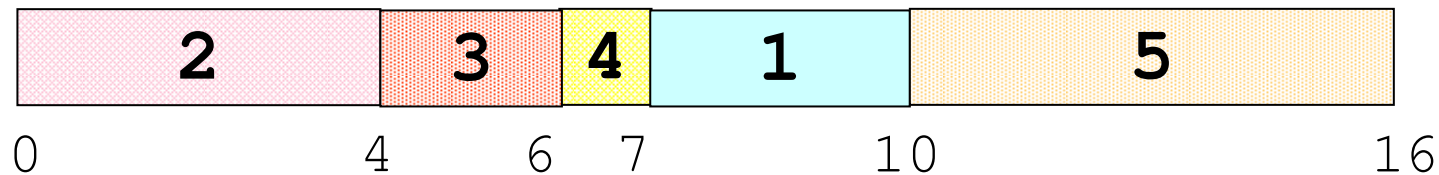
Nodo { 2 3 - - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

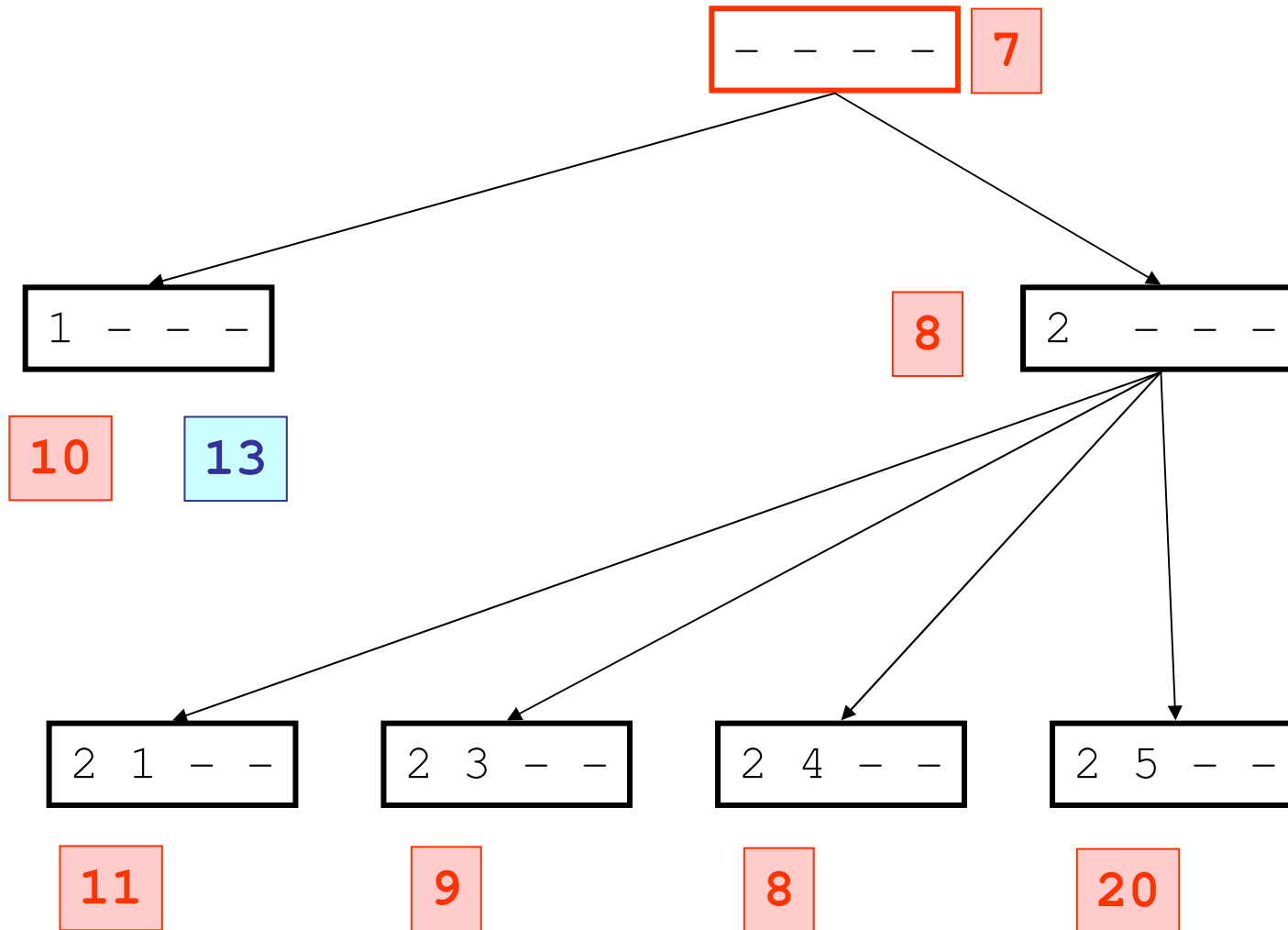
SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 43 - 34 = 9$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 0 + 2 + 8 = 15$$

Branching



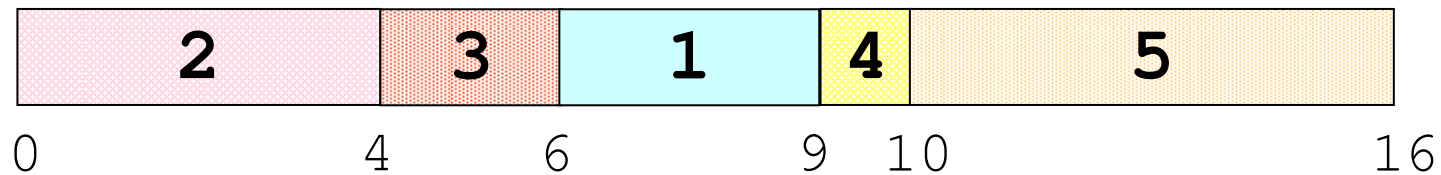
eliminato per bound.

Nodo {2 3 1 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 45 - 34 = 11$$



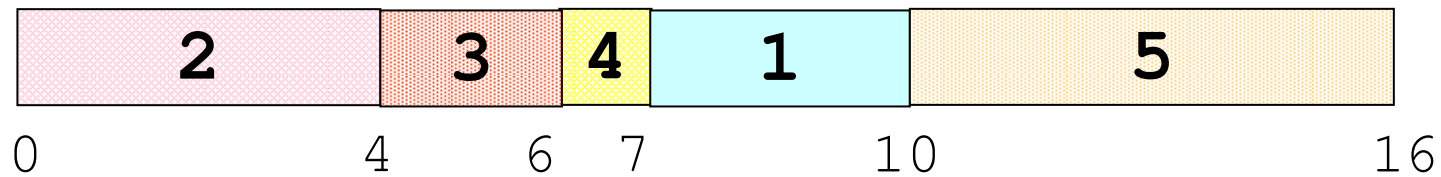
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 4 + 0 + 0 + 5 + 8 = 17$$

Nodo {2 3 4 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 43 - 34 = 9$$



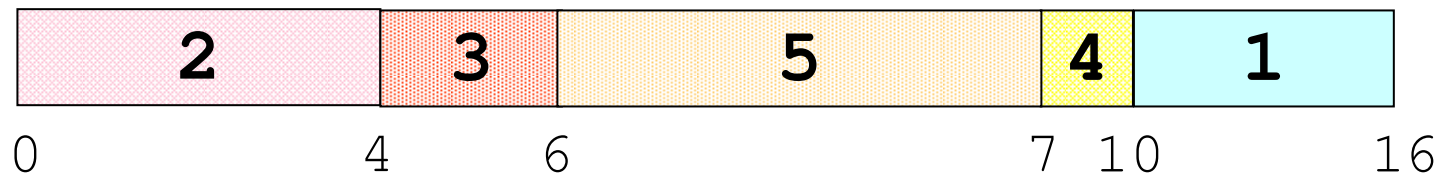
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 0 + 2 + 8 = 15$$

Nodo {2 3 5 - -}

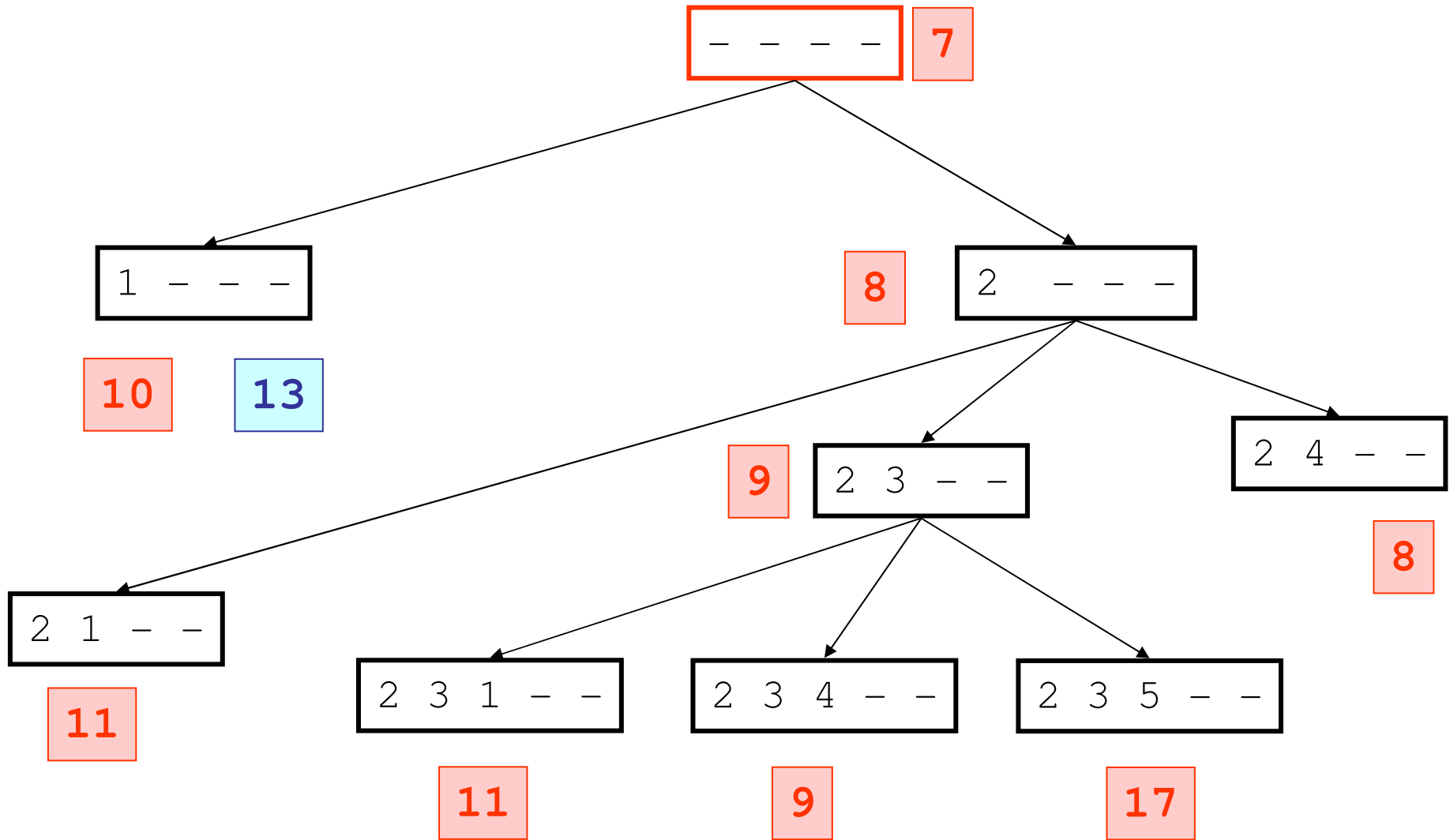
job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 51 - 34 = 17$$



Branching



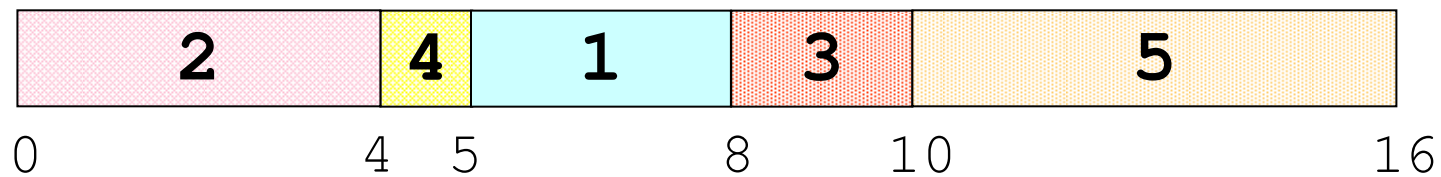
eliminato per bound

Nodo { 2 4 1 - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 43 - 34 = 9$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 3 + 0 + 4 + 0 + 8 = 15$$

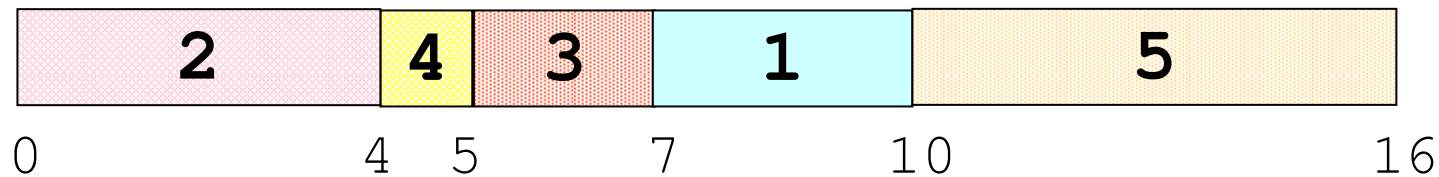
Nodo { 2 4 3 - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 42 - 34 = 8$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 1 + 0 + 8 = 14$$

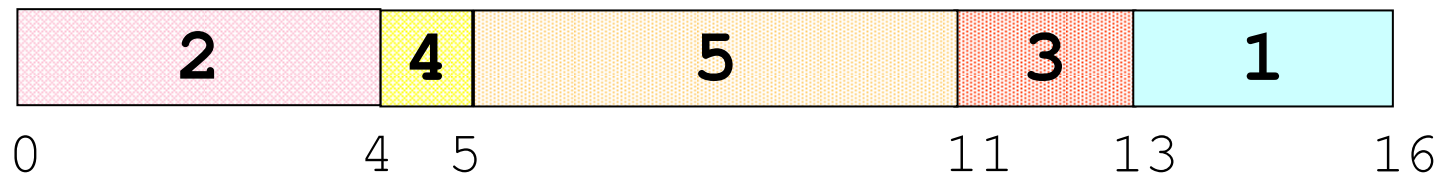
Nodo { 2 4 5 - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

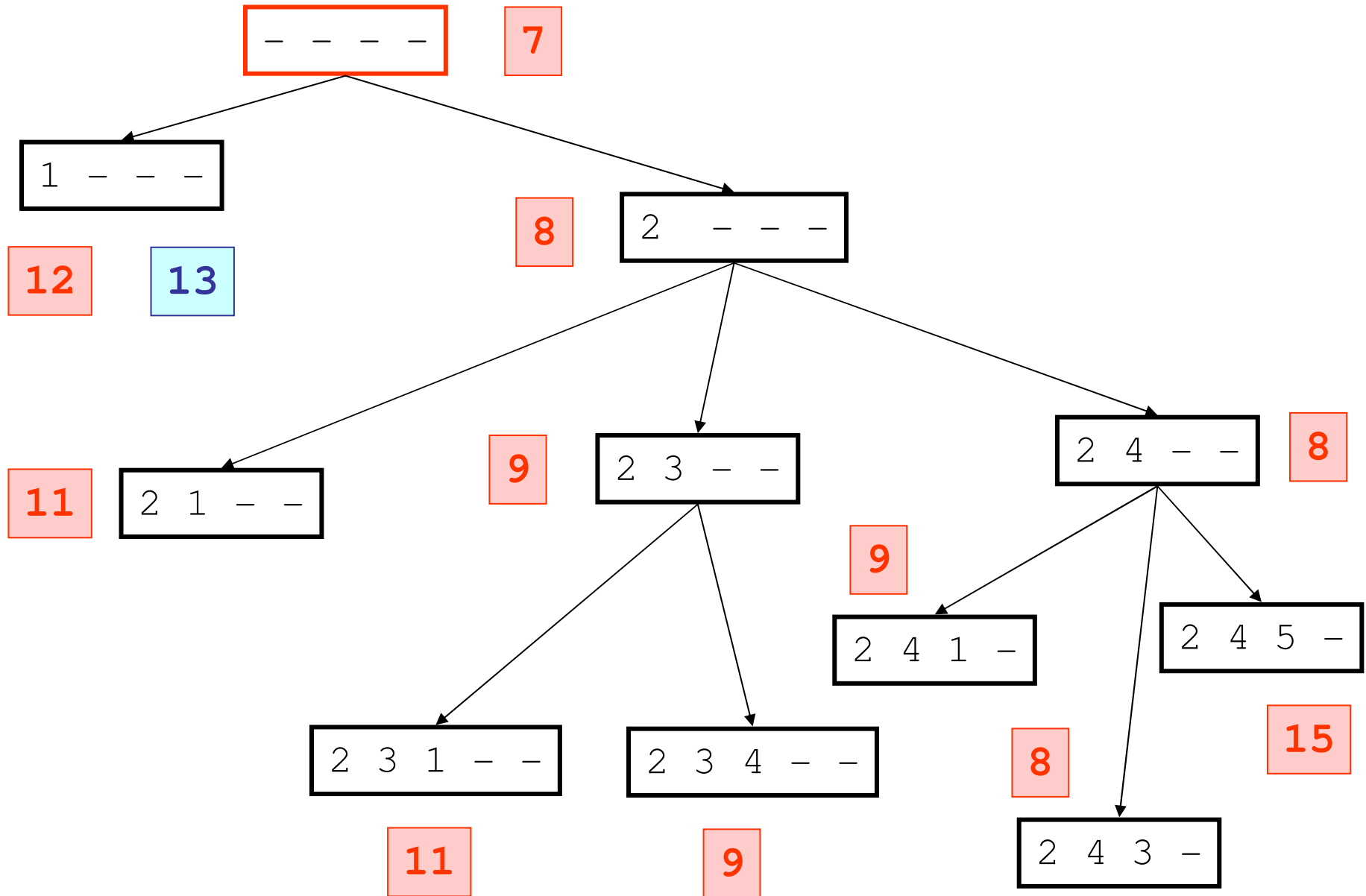
$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 49 - 34 = 15$$



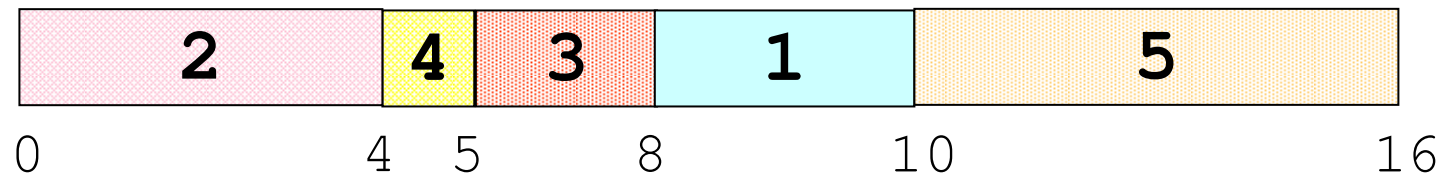
Branching



Nodo {2 4 3 1 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

SRPT costruisce una soluzione ammissibile

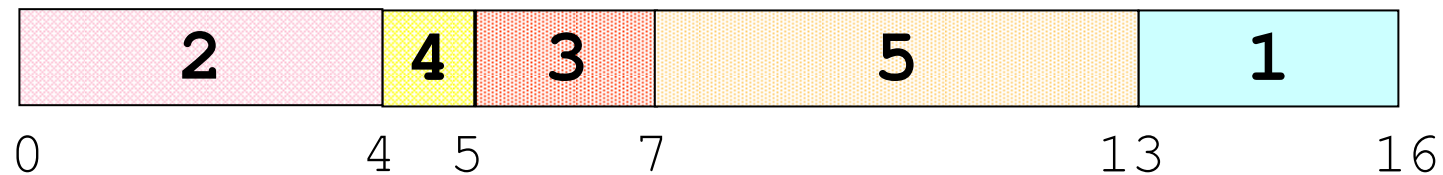


$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 1 + 0 + 8 = 14$$

Nodo {2 4 3 5 1}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

SRPT costruisce una soluzione ammissibile

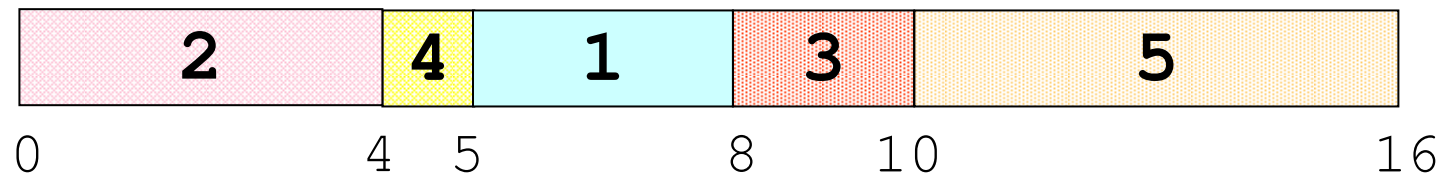


$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 11 + 0 + 1 + 0 + 8 = 20$$

Nodo {2 4 1 3 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

SRPT costruisce una soluzione ammissibile

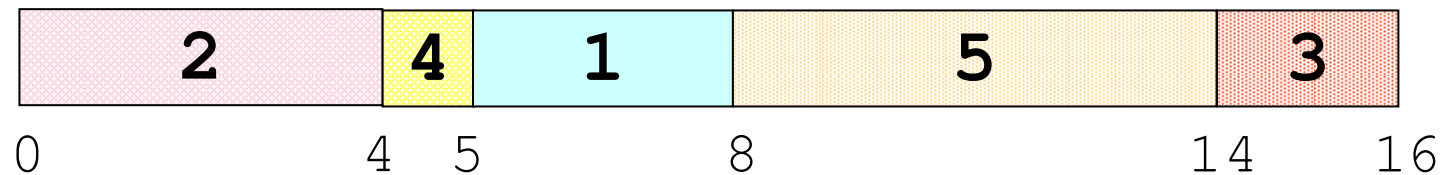


$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 3 + 0 + 4 + 0 + 8 = 15$$

Nodo {2 4 1 5 3}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

SRPT costruisce una soluzione ammissibile



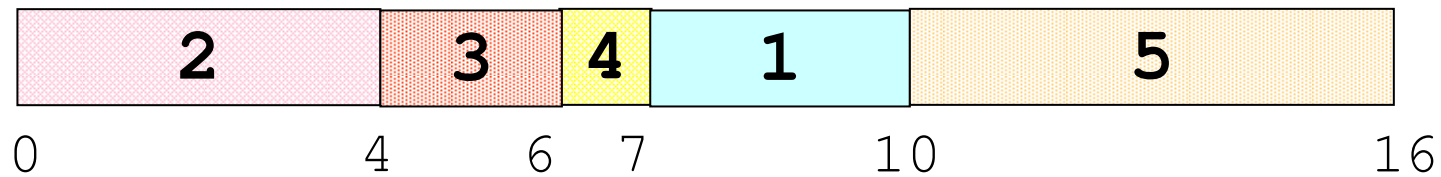
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 3 + 0 + 10 + 0 + 8 = 21$$

Nodo {2 3 4 1 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 43 - 34 = 9$$



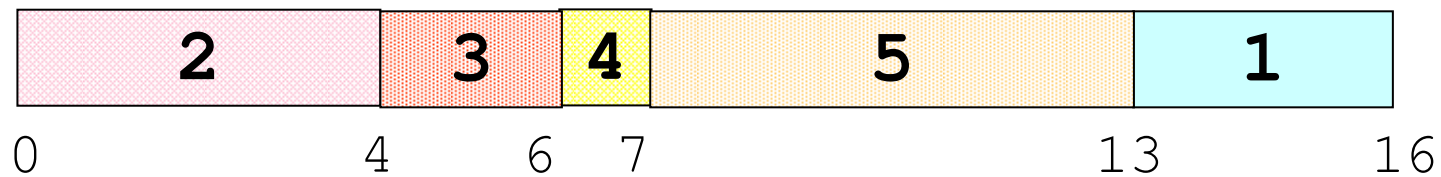
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 0 + 2 + 8 = 15$$

Nodo {2 3 4 5 1}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

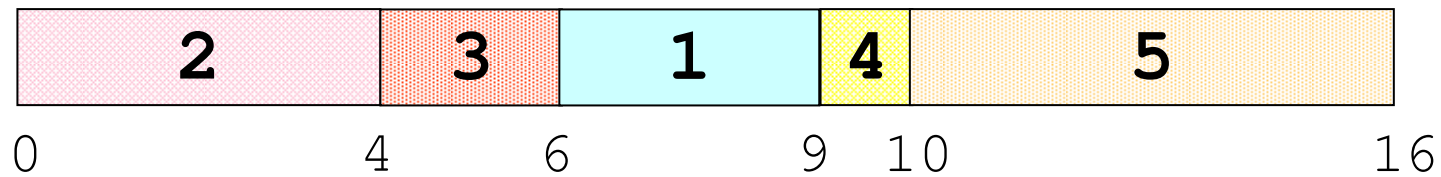
$$\Sigma L_j = 43 - 32 = 11$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 11 + 0 + 0 + 1 + 8 = 20$$

Nodo {2 3 1 4 5}

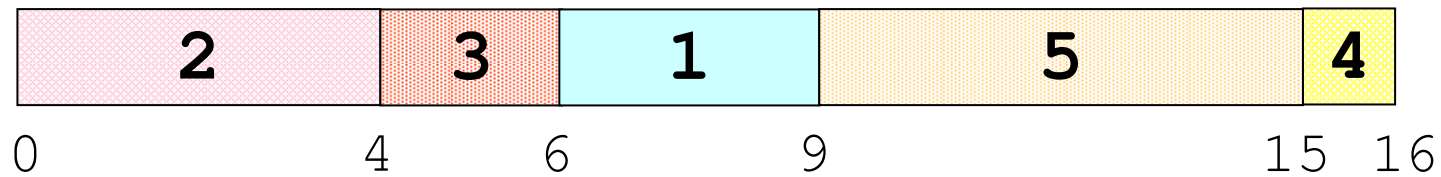
job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 4 + 0 + 0 + 5 + 8 = 17$$

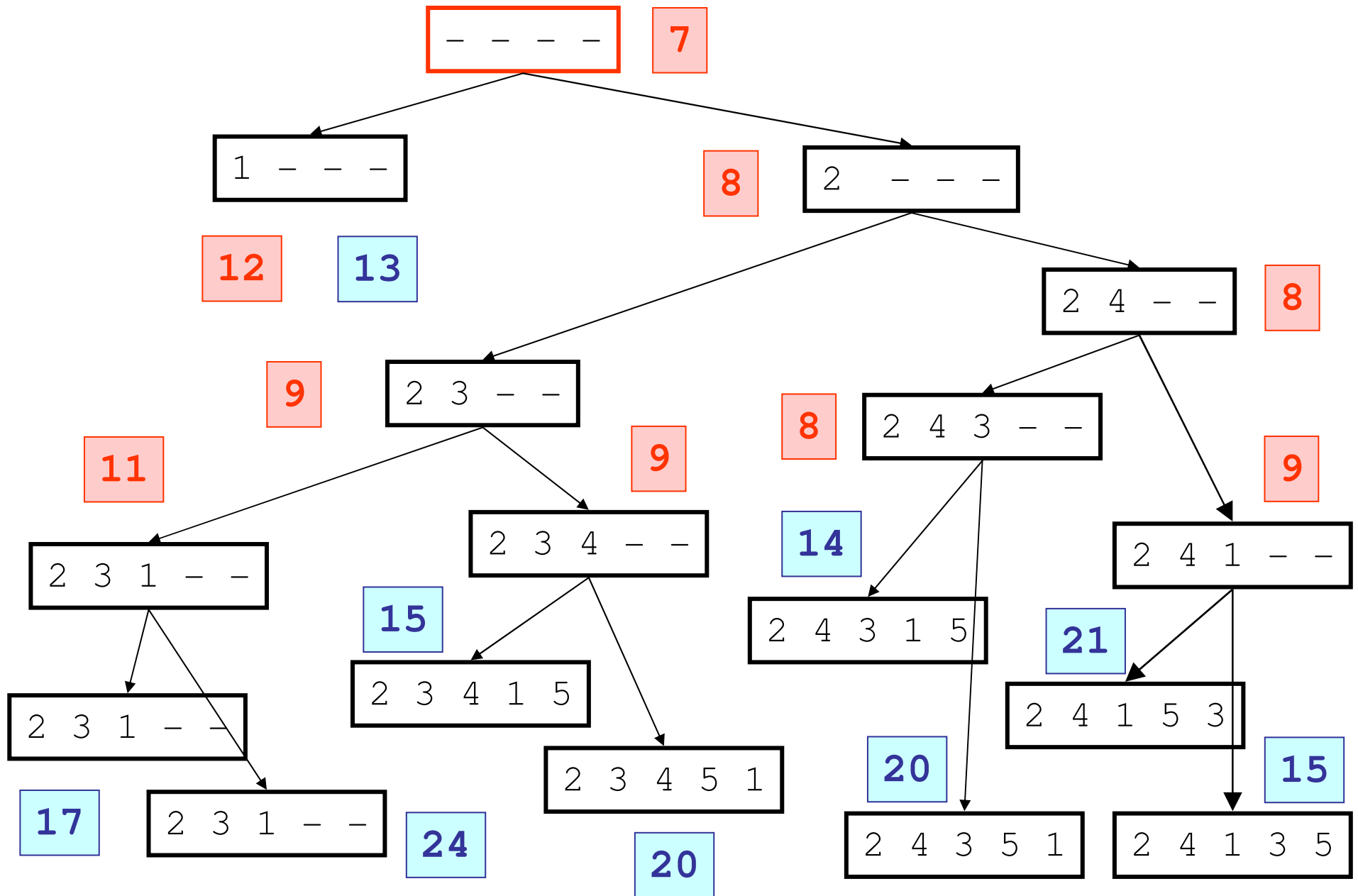
Nodo {2 3 1 5 4}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 5 + 0 + 0 + 11 + 8 = 24$$

Branching

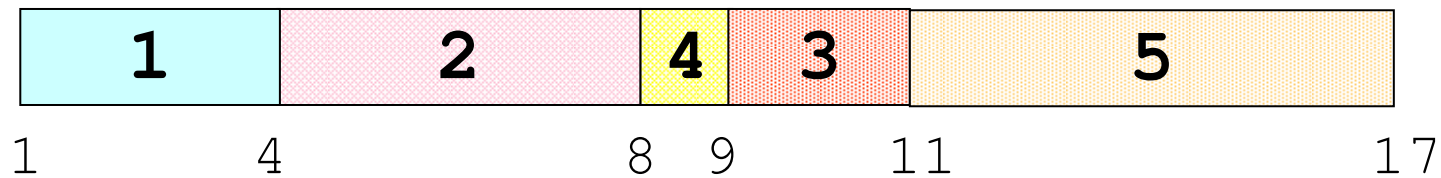


Nodo {1 2 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 49 - 34 = 15$$

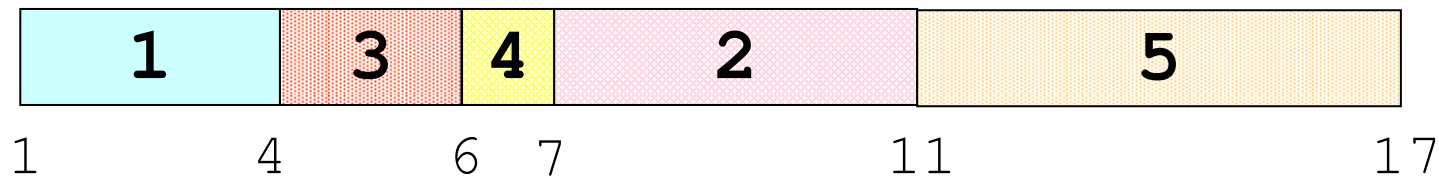


Nodo {1 3 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 45 - 32 = 11$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 5 + 1 + 2 + 8 = 15$$

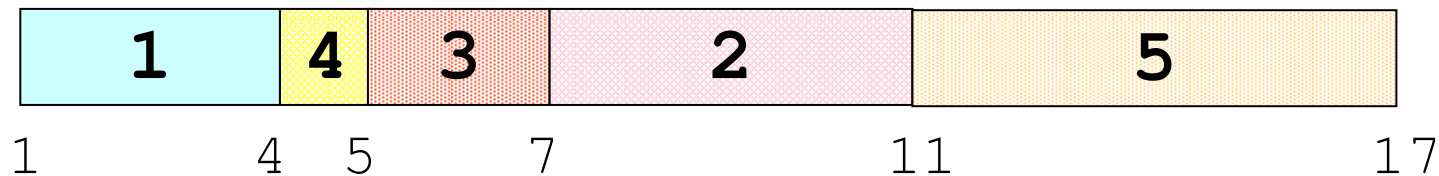
Nodo {1 4 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 44 - 34 = 10$$



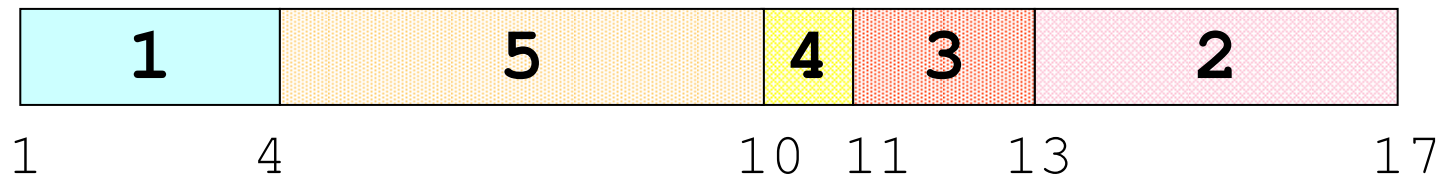
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 5 + 1 + 0 + 8 = 13$$

Nodo {1 5 - - -}

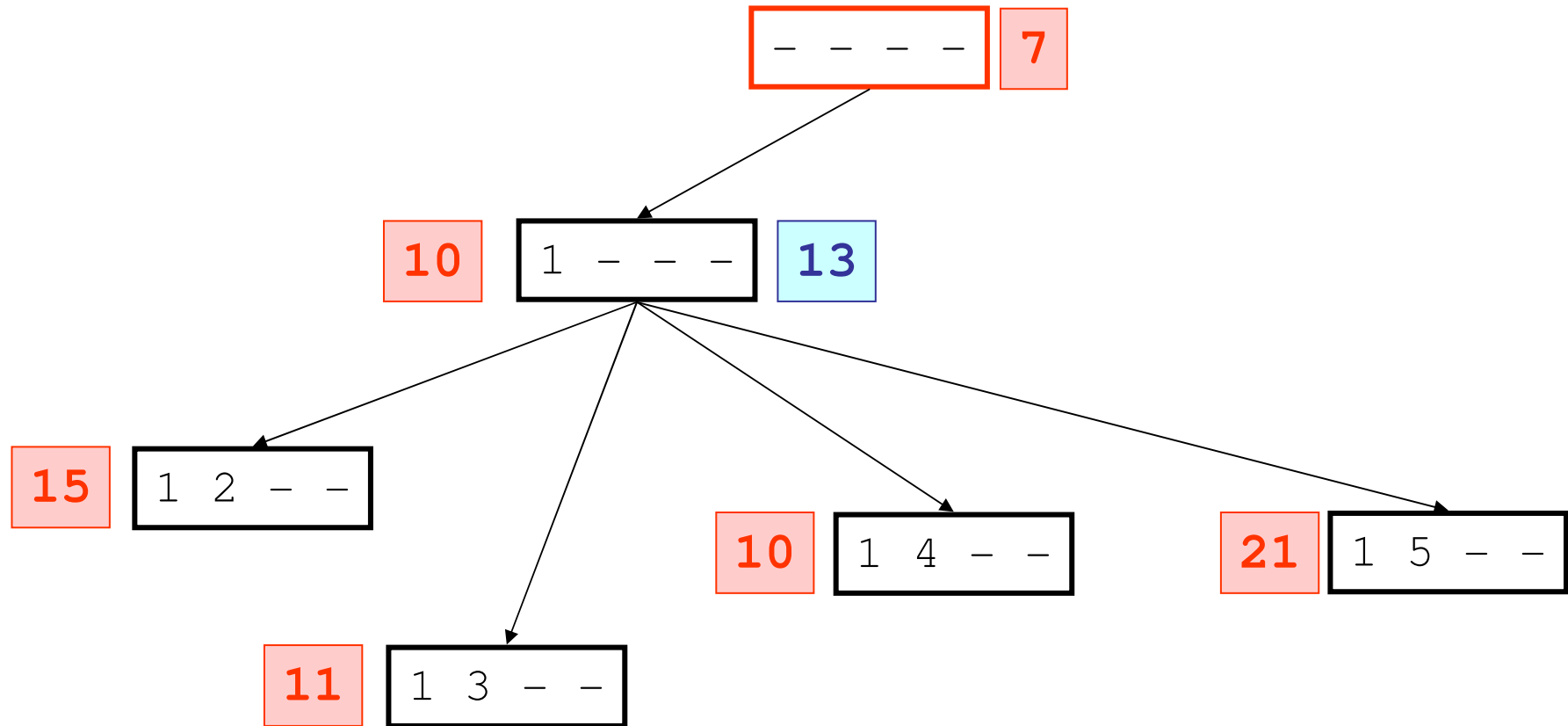
job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 55 - 34 = 21$$



Branching

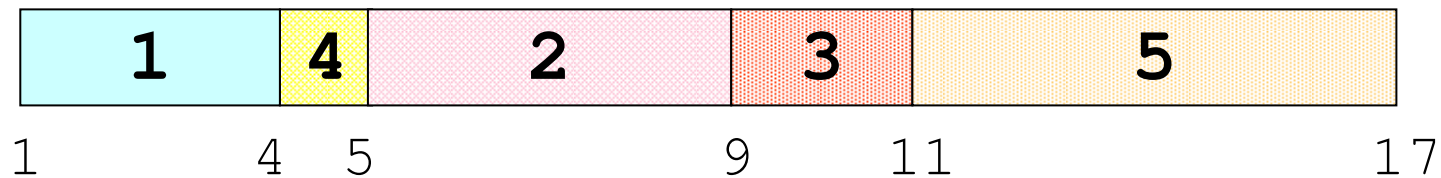


Nodo {1 4 2 -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 46 - 34 = 12$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 3 + 5 + 0 + 8 = 15$$

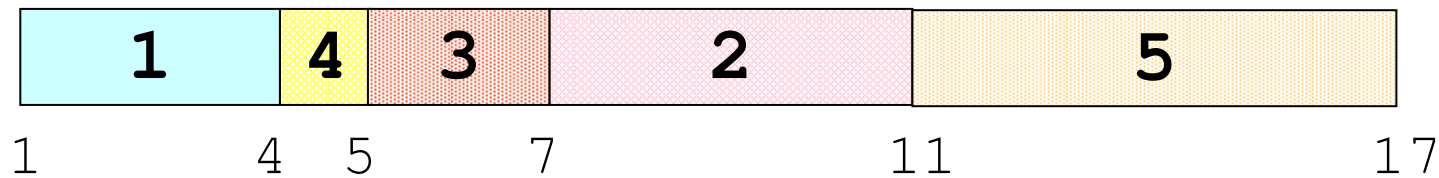
Nodo {1 4 3 -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 44 - 34 = 10$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 5 + 1 + 0 + 8 = 13$$

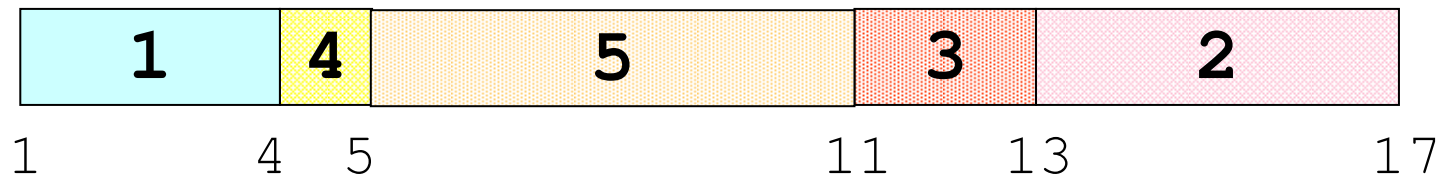
Nodo {1 4 5 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

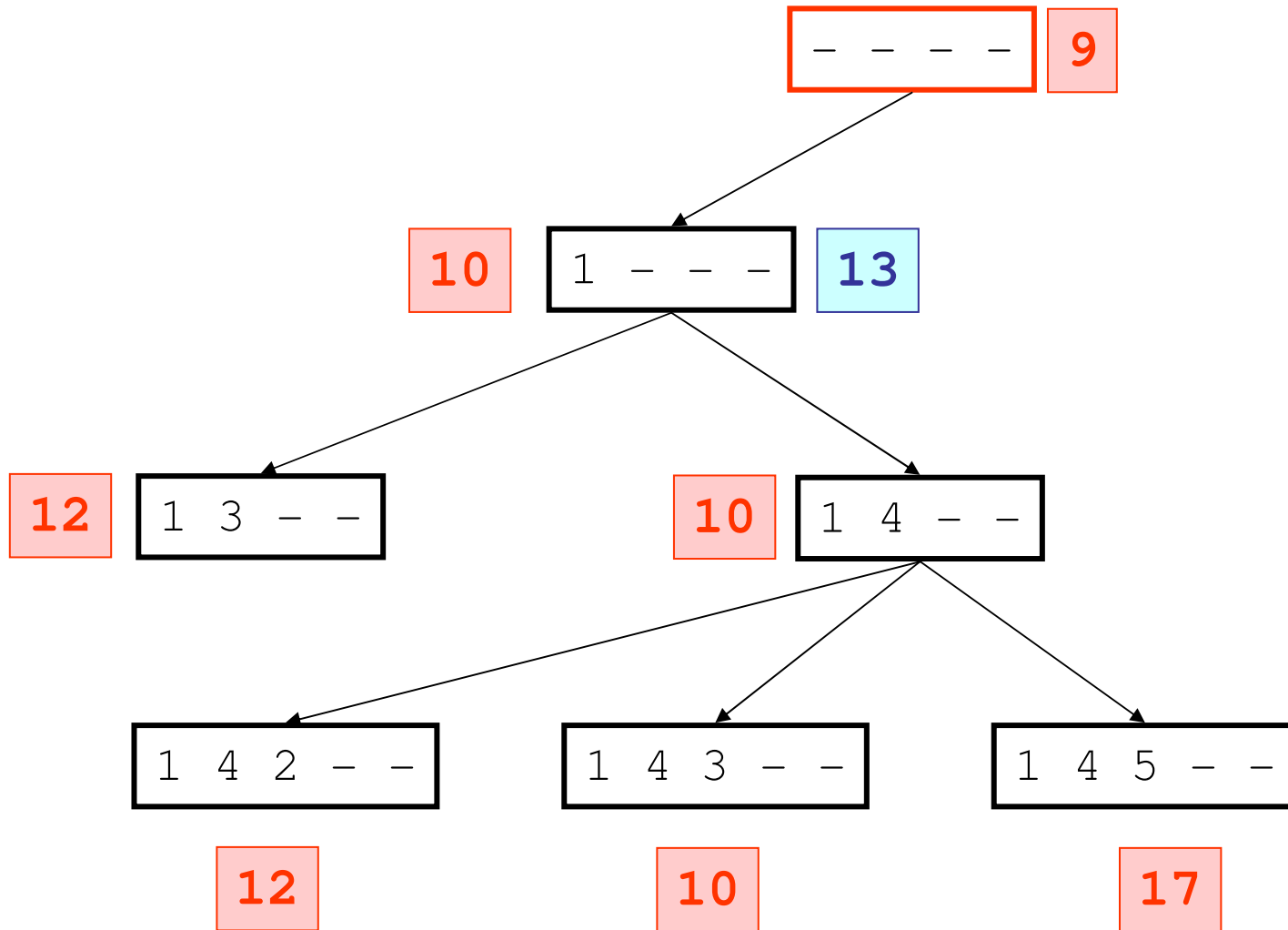
$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 50 - 34 = 16$$



Branching



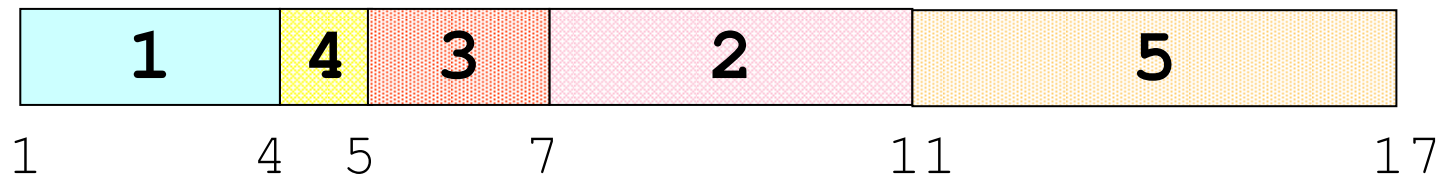
eliminato per bound

Nodo {1 4 3 2 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\sum d_j = 34$$

SRPT costruisce una soluzione ammissibile



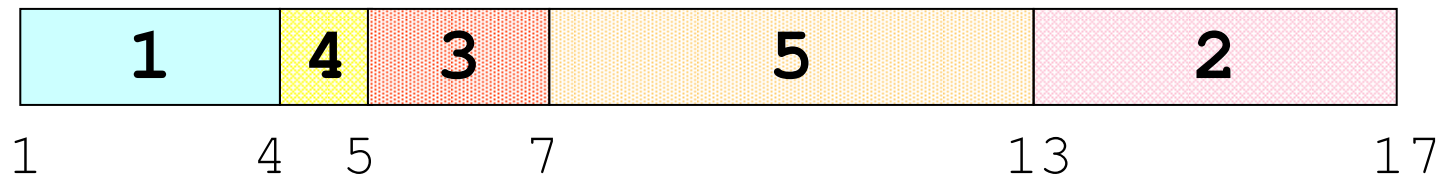
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 5 + 1 + 0 + 8 = 13$$

Nodo {1 4 3 5 2}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

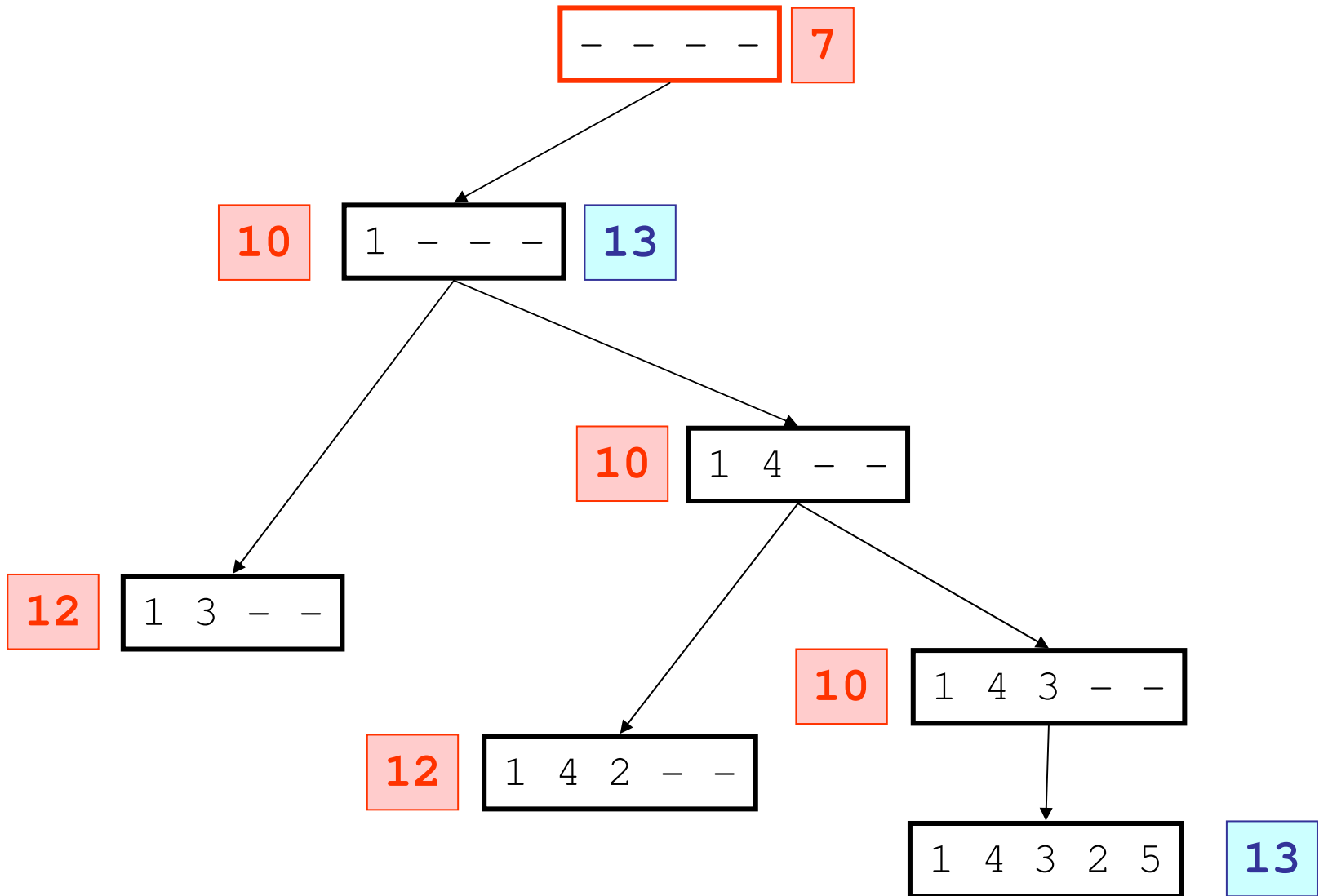
$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 11 + 1 + 0 + 8 = 19$$

Branching

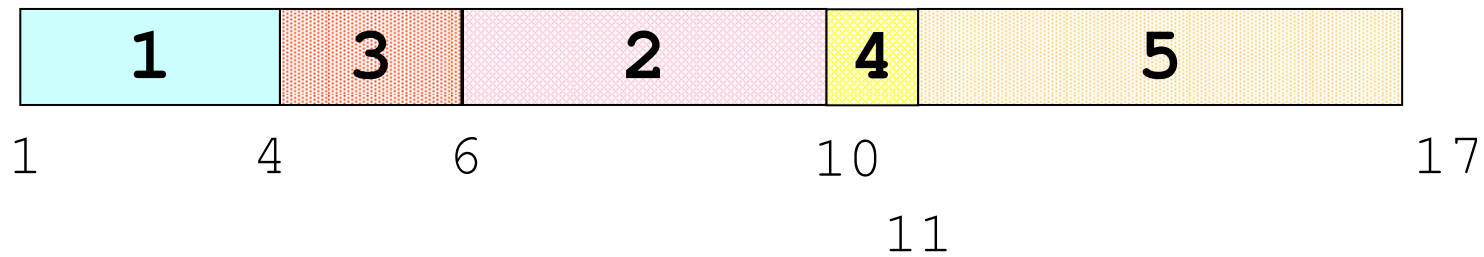


Nodo {1 3 2 -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 48 - 34 = 14$$

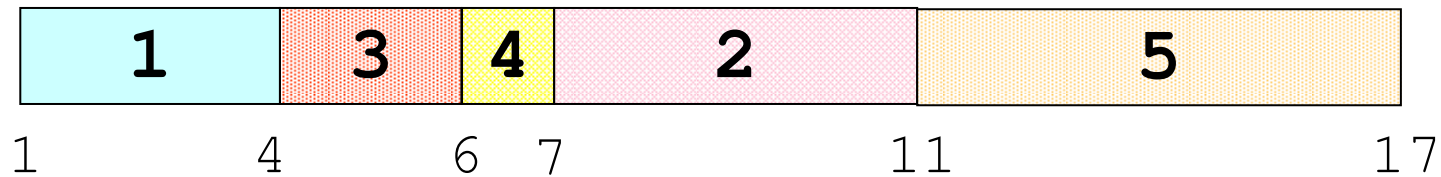


Nodo {1 3 4 -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 45 - 34 = 11$$

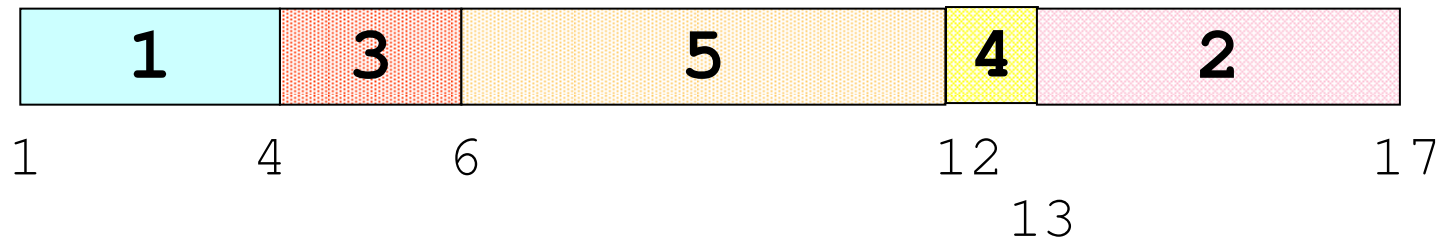


Nodo {1 3 5 -}

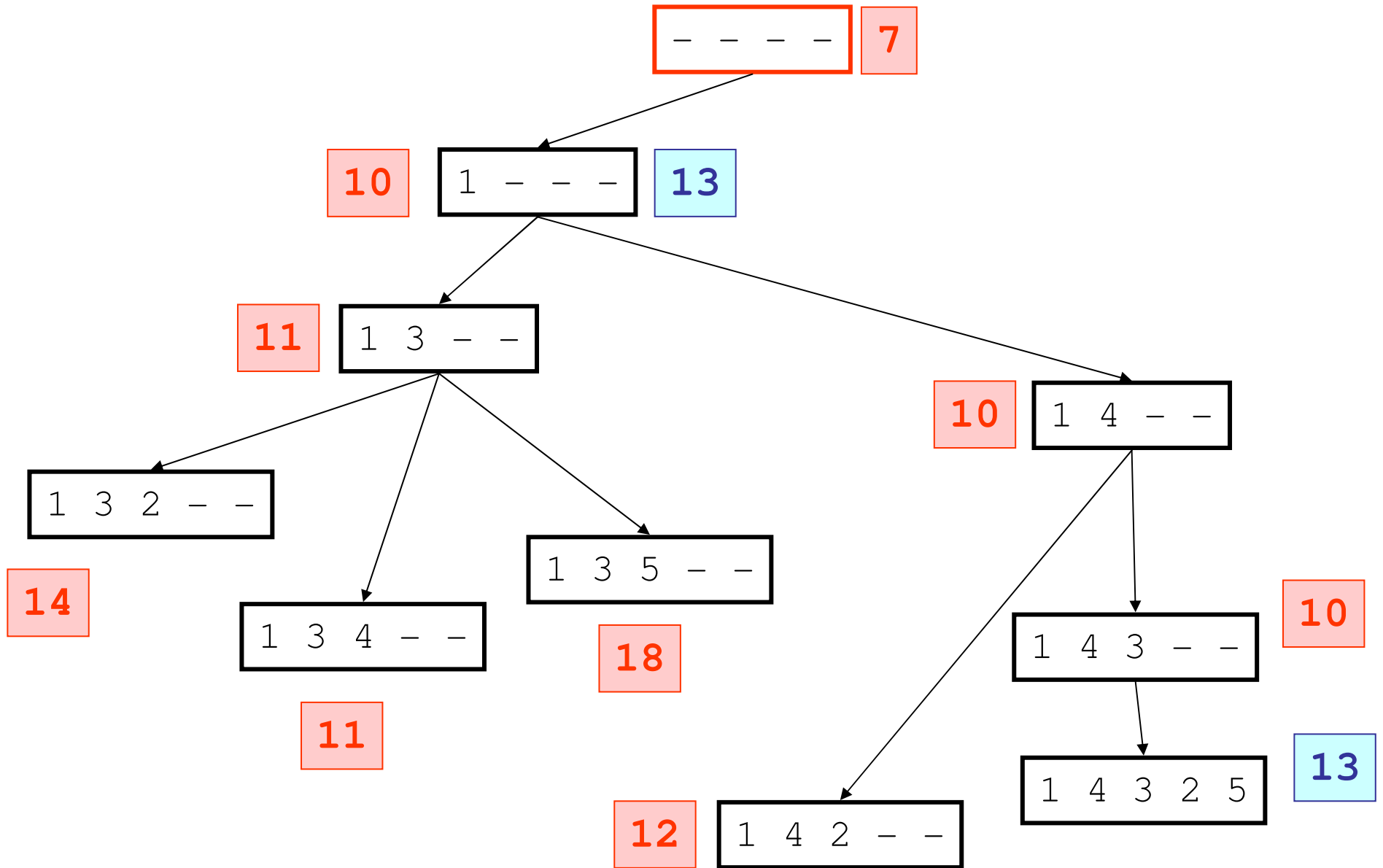
job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

$$\Sigma L_j = 52 - 34 = 18$$



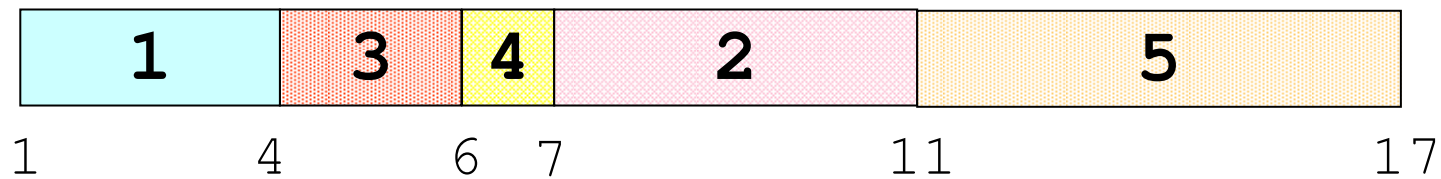
Branching



Nodo {1 3 4 2 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

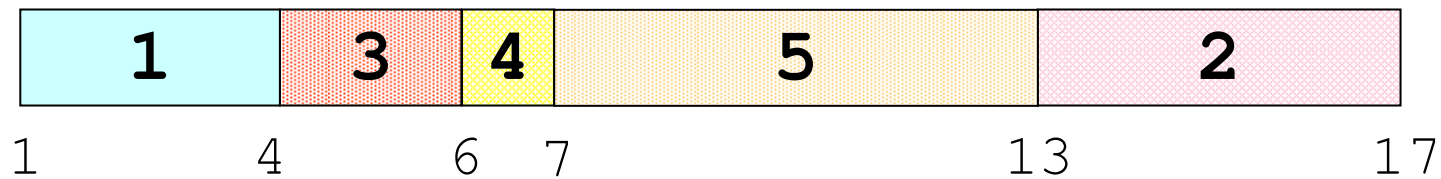
$$\Sigma d_j = 34$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 5 + 0 + 2 + 8 = 14$$

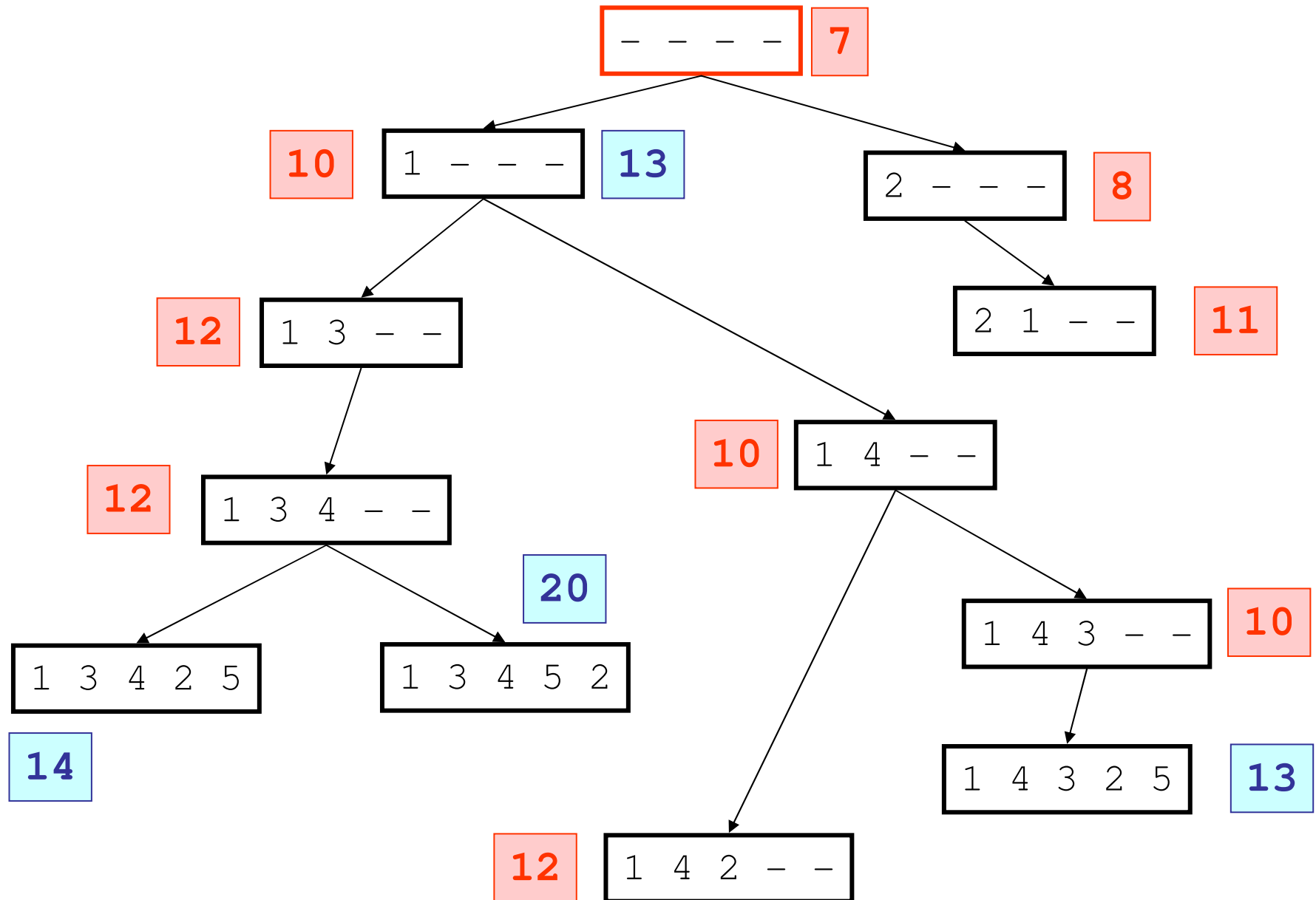
Nodo {1 3 4 5 2}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 11 + 0 + 2 + 8 = 20$$

Albero corrente



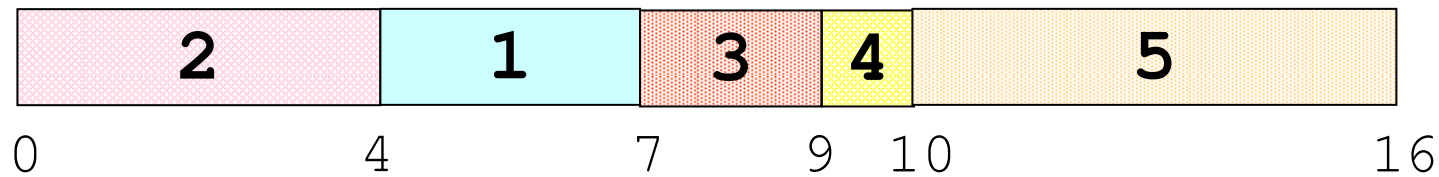
Nodo {2 1 3 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 46 - 34 = 12$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 3 + 5 + 8 = 18$$

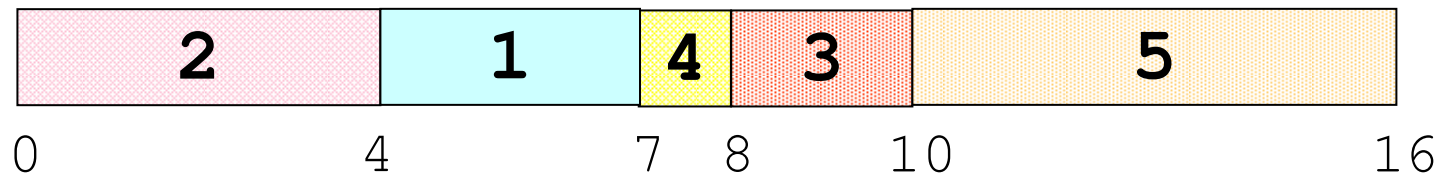
Nodo { 2 1 4 - - }

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 45 - 34 = 11$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 4 + 3 + 8 = 17$$

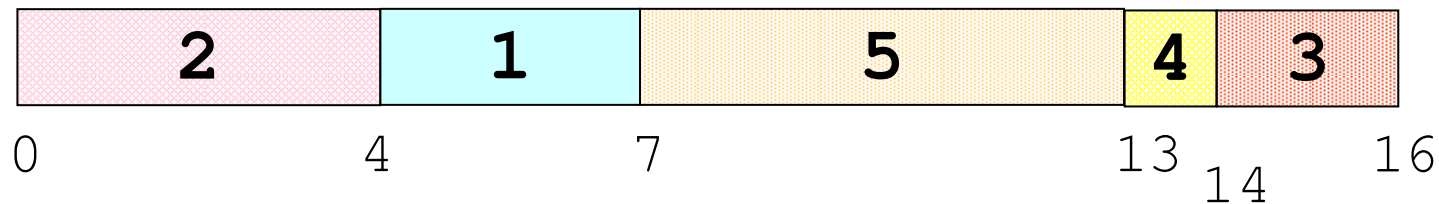
Nodo {2 1 5 - -}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

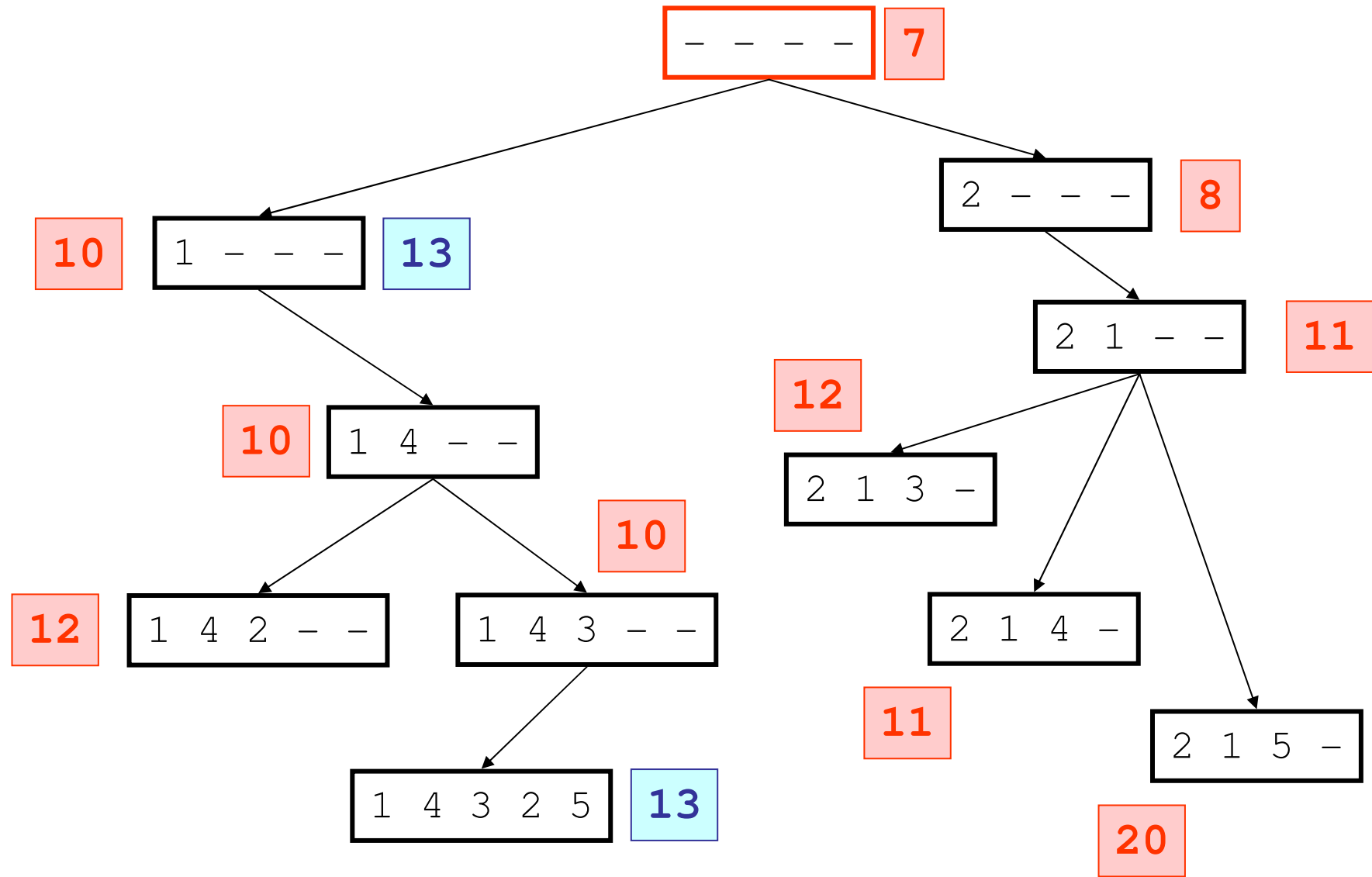
SRPT costruisce una soluzione ammissibile

$$\Sigma L_j = 54 - 34 = 20$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 10 + 9 + 8 = 29$$

Albero corrente

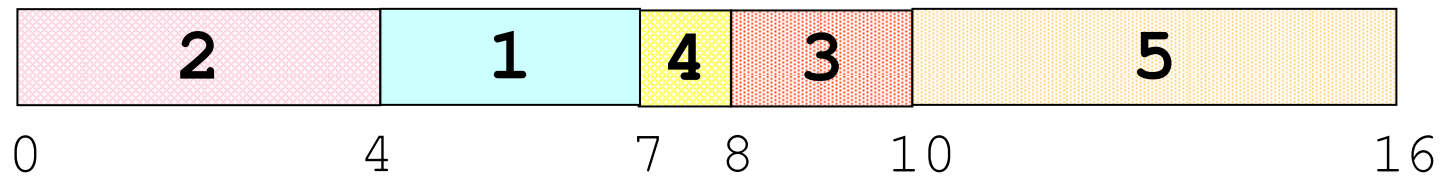


Nodo {2 1 4 3 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile



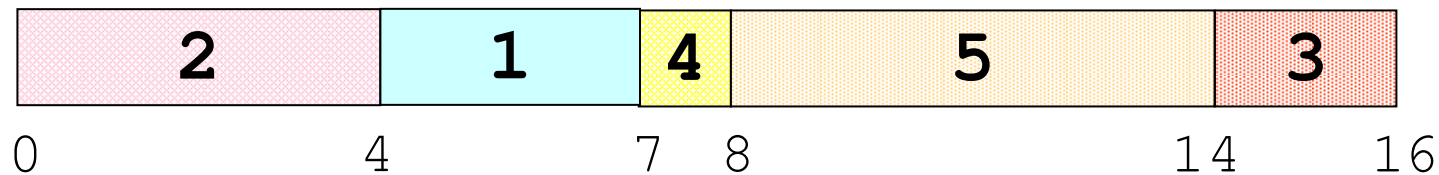
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 4 + 3 + 8 = 17$$

Nodo {2 1 4 5 3}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile



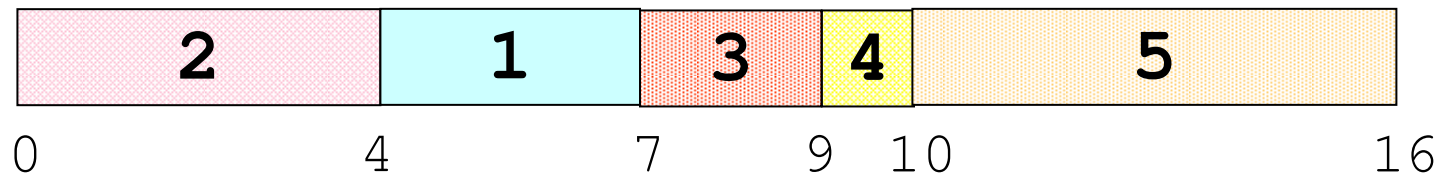
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 10 + 3 + 8 = 23$$

Nodo {2 1 3 4 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile



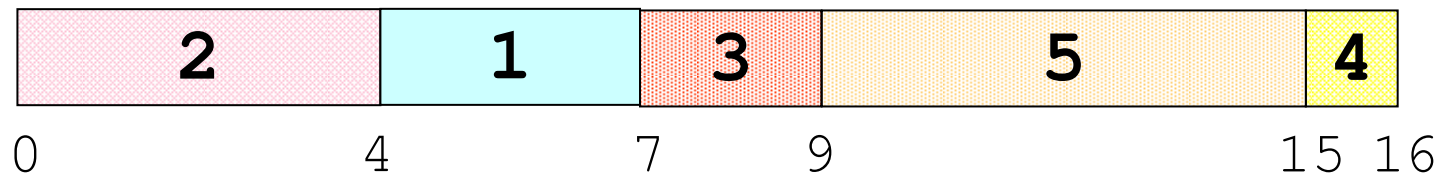
$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 3 + 5 + 8 = 18$$

Nodo {2 1 3 5 4}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

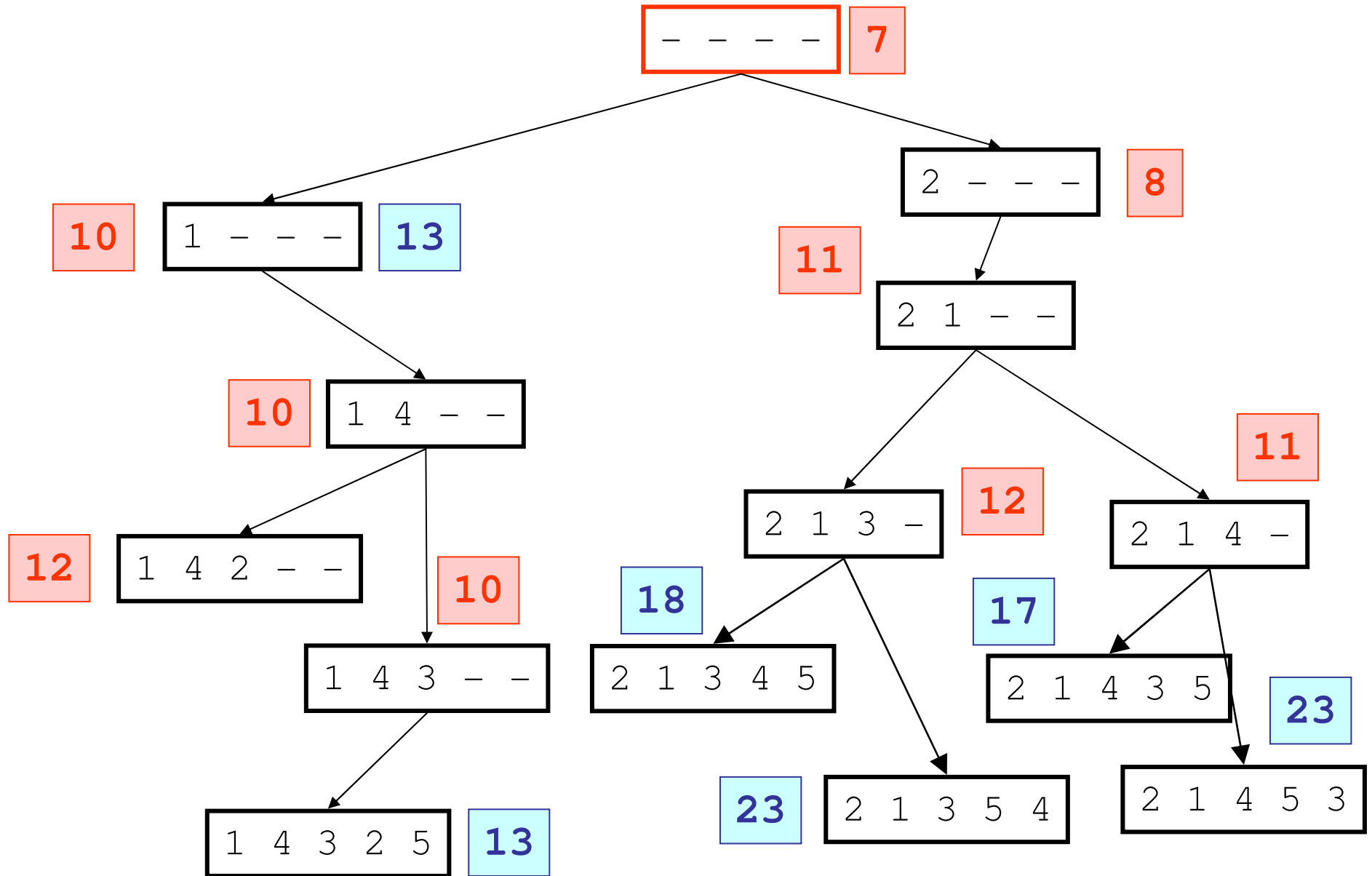
$$\Sigma d_j = 34$$

SRPT costruisce una soluzione ammissibile



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = 2 + 0 + 3 + 10 + 8 = 23$$

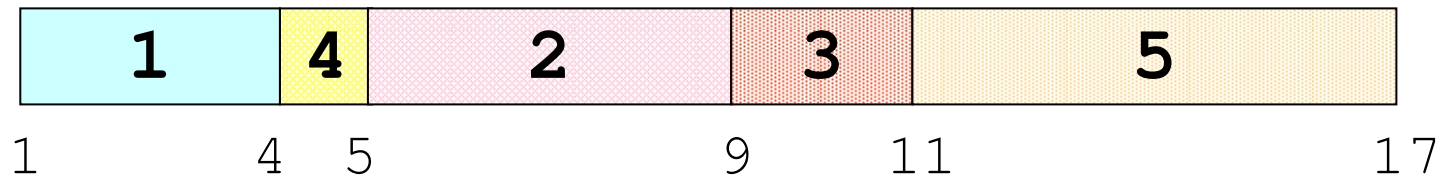
Albero corrente



Nodo {1 4 2 3 5}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$

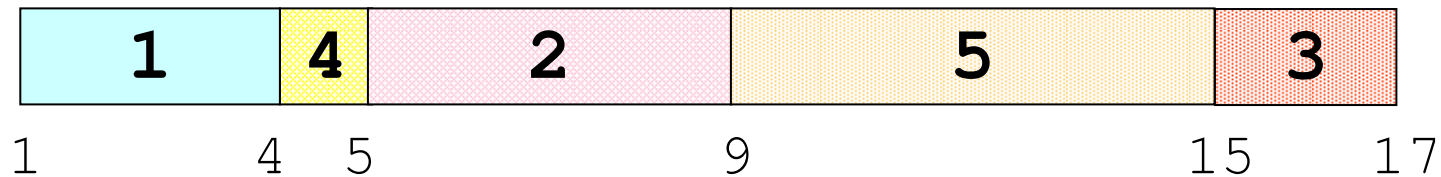


$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 3 + 5 + 0 + 8 = 15$$

Nodo {1 4 2 5 3}

job	1	2	3	4	5
p_j	3	4	2	1	6
r_j	1	0	4	2	3
d_j	5	6	6	5	12
h_j	L_1	T_2	T_3	L_4	$8U_5$

$$\Sigma d_j = 34$$



$$L_1 + T_2 + T_3 + L_4 + 8U_5 = -1 + 3 + 11 + 0 + 8 = 21$$

Albero corrente

